Table

Description automatically generated

Let’s denote the amount of raw ingredients as follows: cream () , egg yolk () , whole milk () , frozen sweetened egg yolk () , glucose syrup (), water ()

This problem as a linear program is:

minimize   
subject to   
   
   
   
 

Table

Description automatically generated

The standard form of LP is

maximize   
subject to   
   
This can be done by introducing slack variables. The standard form of the LP in part (a) becomes:

maximize   
subject to   
   
   
   
   
However, we can notice that  and serves the same purpose as the slack variable in the last equality, there is no need to introduce for this condition. The standard form becomes

maximize   
subject to   
   
   
   
 

**Chart

Description automatically generated**

**Text

Description automatically generated** Saturated edges are those where its flow equals to its capacity. Therefore, in this definition, the saturated edges are (s, a), (d, a), (d, c) and (c, t).

**Text

Description automatically generated**

The residual graph of the flow graph is:

A picture containing sky

Description automatically generated

(Graph drawn at online application <http://graphonline.ru/en/?graph=DpzftrIbcfuAuPwh>)

**Text

Description automatically generated**

In the residual graph in part (b), the source vertex s has only one outcoming edge, which goes to vertex d. From d, there are only incoming edges, which means that the vertex s and t has been disconnected in the residual graph. In other words, there are no augmented paths P that connects vertices s and t in the residual graph

* The given flow is a maximum flow

**Graphical user interface, text

Description automatically generated**

**A screenshot of a computer

Description automatically generated with medium confidence**

The relaxation of this integer program

minimize 

subject to 



**A screenshot of a computer

Description automatically generated with medium confidence**

When we take close look at this condition,, it means that the sum of the set value where e is contained in must be nonzero. Between 0 and 1, only 1 is nonzero. Because this problem has been turned into a fractional problem, the sum of at most 3 sets must be larger than or equal to 1. Proof by contradition:

Supposed that e is contained in 3 different sets of s and all of their sets value is strictly smaller than 1/3. Then , which fails to satisfy the condition 

* There must exists one set  such that  to ensure that the condition is guaranteed. This proof also applies when the number of sets that e is contained in is less than 3 sets.

**A screenshot of a computer

Description automatically generated with medium confidence**

The solution of the original IP problem is at most 3 times larger than the optimum solution (3-approximation) of the relaxed LP because the cost of the relaxed LP increased by a factor of 3, and we know that at least one set whose value  is included to cover all elements. Altogether, the cost of the relaxed LP is at least 3 times smaller than the cost of the initial IP.

**A screenshot of a computer

Description automatically generated with medium confidence**From the OPT solution of the relaxed LP, the feasible solution for the initial IP can be constructed as follows:

1. Generate a fractional value from a black box. This value must be at least 1/3. Then assign it to a random set where it still has some elements inside
2. Record which elements e are contained inside this set s. Then cross out this element from other sets where e is contained it (at most 2 sets)
3. Repeat step (1-2) until no elements are left that have not been contained in any sets
4. Assign 0 to all the rest sets.
5. Finally, multiply the value of all sets s by 3. This is the final solution of the LP problem.